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Realization of Fractance Device using Fifth Order Approximation

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ABSTRACT
The realization of Fractance device is an important topic of research for the people working in Fractional Calculus, control systems, signal processing, and other allied fields. Having multifaceted applications, the realization of the device has gained importance from the past few years. The important step in the realization of Fractance device is finding the rational approximation that best fits its behavior. In this paper, the rational approximation is calculated using the continued fraction expansion formula. The rational approximation thus obtained is synthesized as a passive circuit using MATLAB. The active circuit is obtained by making use of the Operational Amplifier. The passive active circuits are simulated using TINA-TI software. The working of the proposed circuits is studied. It has been observed that both theoretical and simulated results match each other.

General Terms
Continued Fraction Expansion, Rational Approximation, Fractional order systems, Fractance device

Keywords
Fractional order, Resistance, Capacitance, Active circuit, Passive Circuit, Realization, Phase response

1. INTRODUCTION
Fractional calculus deals with the differentiation and integration to an arbitrary order[1-2]. The equation which contains fractional order differentiation and integration is called as Fractional order Equation. Any system which is defined by the Fractional order differential equation is called as a Fractional order system. Transmission lines, Diffusion of heat into solids, $PI^{\lambda}D^{\mu}$ controllers are some of the examples of Fractional order systems [3,4]. Some of the possible applications of the Fractional Calculus are discussed in detail in [3].

Fractance device is also an example of Fractional order System. The device is defined by the impedance which is proportional to $1/s^{\alpha}$, where α is a fractional order. As the value of the α changes, the behavior of the fractance changes. At $\alpha=0$, it behaves as a resistor. When α changes from -1 to +1 its behavior changes from inductor to capacitor. It acts as a Frequency dependent Negative Resistor (FDNR) when $\alpha=-2$[9,10]. Since a mixed behavior is present in a single element the realization of the element has gained interest. For an ideal Fractance device the phase angle is constant independent of the frequency range of operation. But the phase angle depends only on the value of fractional order. So fractance device is also called as *constant phase angle device* or simply *fractor[11,12,13]*. In [17], Karabi Biswas et.al has proposed a commercially available fractance device and studied its operation and performance with respect to a differentiator circuit. This is preliminary attempt and the commercially available device need to be yet investigated. Fractional order Lowpass, Highpass and other types of filters are also possible with Fractance device[14,15,16,17]. Time domain response calculations of the Fractance device are presented in [18].

The realization of Fractance is possible with passive elements connected in different manners. Fractance device can be of tree type or chain type .M.Nakagawa and K.Sorimachi, have proposed a circuit consisting of self-similar tree type circuit with resistors and capacitors [34].Oldham and spanier have realized fractance circuit using *N* pairs of *RC* connected as chain network [16] .Recently a net grid type circuit for the realization of fractance device is proposed[16] .

The key point in the realization of fractance device is finding the rational approximation of the fractional order operator. There are many of the procedures exist for the calculation of rational approximations in both time and frequency domain. Oustaloup approximation, Carlson approximation, Mastuda Approximation, are some of the prominently used approximation techniques for the calculation of the rational approximation[5,6,7,19,21,22]. Even though there are several approximations possible, they are having their own advantages and disadvantages. In the Literature, in 2011 rational approximation using Continued Fraction Expansion method is proposed[21]. The rational approximation for $\alpha=1/2$, is presented in [21]. It is realized using operational amplifiers. But the realizations for $\alpha=1/3,1/4$ and other fractional orders are not yet realized. In this paper an attempt is made to realize fractional order operator or Fractance for $\alpha=-1/3,-1/4$ and its operation is verified in both time and frequency domains. A spice based software TINA is used for the simulation purpose.

The paper is organized as follows. In Section 2, the derivation of the rational approximation is presented. The realization using partial fraction expansions is presented in Section 3. Section 4 deals with the Active realization of the circuit using TINA-TI software. Finally conclusions are drawn in Section 5.

2. RATIONAL APPROXIMATION
In this section, a novel rational approximation proposed in [21] is studied for various values of fractional order α. This proposed approximation is based on continued fraction expansion formula. It is given as[23,21],

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$$(1+x)^{\alpha} = \cfrac{1}{1-\cfrac{\alpha x}{1+\cfrac{1}{2}\cfrac{(\alpha+1)x}{1-\cfrac{1}{6}\cfrac{(\alpha-1)x}{1+\cfrac{1}{6}\cfrac{(\alpha+2)x}{1-\cfrac{1}{10}\cfrac{(\alpha-2)x}{1+\cfrac{1}{10}\cfrac{(\alpha+3)x}{1-\cfrac{1}{14}\cfrac{(\alpha-3)x}{1+\ \dots\dots\dots}}}}}}}} \qquad (1)$$

This continued fraction expansion is convergence in the finite complex s-plane from $x = -\infty$ to $= -1$. For two number of terms of the expansion, first order rational order approximation is possible. It can also be observed that to have higher range of values of operation, higher order approximations are to be chosen. Substituting x by $(s-1)$ and taking first ten terms of the series we obtain the truncated rational approximation expansion of s^{α} which is given as,

$$s^{\propto} = \frac{P_0S^5+P_1S^4+P_2S^3+P_3S^2+P_4S+P_5}{Q_0S^5+Q_1S^4+Q_2S^3+Q_3S^2+Q_4S+Q_5} \qquad (2)$$

Where

$P_0 = Q_5 = -\propto^5 - 15\propto^4 - 85\propto^3 - 225\propto^2 - 274\propto -120$

$P_1 = Q_4 = 5\propto^5 + 45\propto^4 + 5\propto^3 - 1005\propto^2 - 3250\propto -3000$

$P_2 = Q_3 = -10\propto^5 - 30\propto^4 + 410\propto^3 + 1230\propto^2 - 4000\propto -12000$

$P_3 = Q_2 = 10\propto^5 - 30\propto^4 - 410\propto^3 + 1230\propto^2 + 4000\propto -12000$

$P_4 = Q_1 = -5\propto^5 + 45\propto^4 - 5\propto^3 - 1005\propto^2 + 3250\propto -3000$

$P_5 = Q_0 = \propto^5 - 15\propto^4 + 85\propto^3 - 225\propto^2 + 274\propto -120$

It is also to be mentioned that the Eqn.(2) has been proved to be more efficient compared to Oustaloup approximation in [19] for a fractional order of ½.

The values of the coefficients for various values of α is tabulated as in Table.1.The value of α is varied from 0.1 to 0.9 in steps of 0.1.

Table.1. 5th order Rational Approximation coefficients insteps of α as 0.1

α	P_0	P_1	P_2	P_3	P_4	P_5
0.1	149.7365	3.3350e+03	1.2387e+04	1.1588e+04	2.6851e+03	94.7665
0.2	184.5043	3.6901e+03	1.2748e+04	1.1154e+04	2.3902e+03	73.5437
0.3	224.8689	4.0649e+03	1.3078e+04	1.0701e+04	2.1152e+03	55.8741
0.4	271.4342	4.4593e+03	1.3378e+04	1.0230e+04	1.8600e+03	41.3338
0.5	324.8438	4.8727e+03	1.3643e+04	9.7453e+03	1.6242e+03	29.5313
0.6	385.7818	5.3045e+03	1.3873e+04	9.2489e+03	1.4074e+03	20.1062
0.7	454.9746	5.7541e+03	1.4066e+04	8.7435e+03	1.2092e+03	12.7284
0.8	533.1917	6.2206e+03	1.4218e+04	8.2317e+03	1.0290e+03	7.0963
0.9	621.2470	6.7029e+03	1.4330e+04	7.7164e+03	866.1229	2.9360

By selecting α as -1/3 &-1/4 the rational approximations will be given as,

$$s^{-1/3} = \frac{11s^5+440s^4+2288s^3+2860s^2+910s+52}{52s^5+910s^4+2860s^3+2288s^2+440s+11} \qquad (3)$$

$$s^{-1/4} = \frac{209s^5+7315s^4+35530s^3+41990s^2+12597s+663}{663s^5+12597s^4+41990s^3+35530s^2+7315s+209} \qquad (4)$$

The magnitude and phase responses for α = -1/3 &-1/4 are shown in Figs.1,2 and 3 respectively. From the figures it can be observed that the Fractance device works well in low frequency regions.

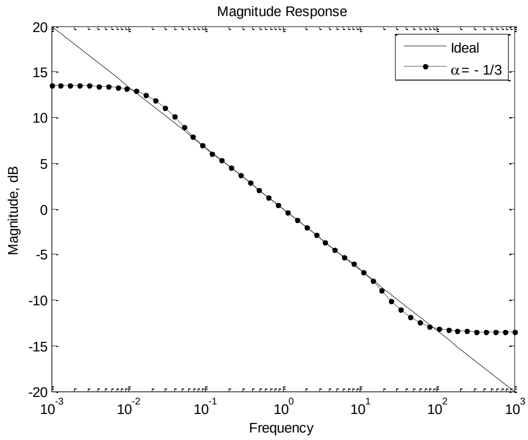

Fig.1.Magnitude response for α=-1/3

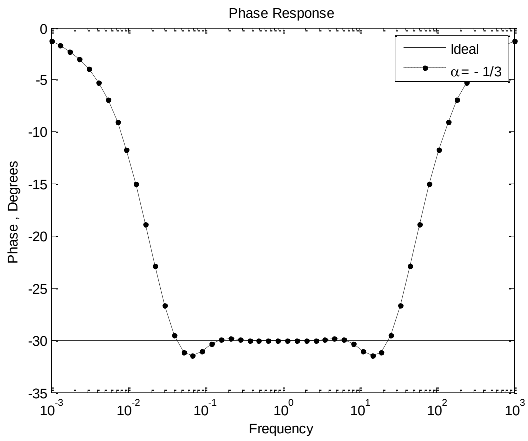

Fig.2.Phase response for α=-1/3

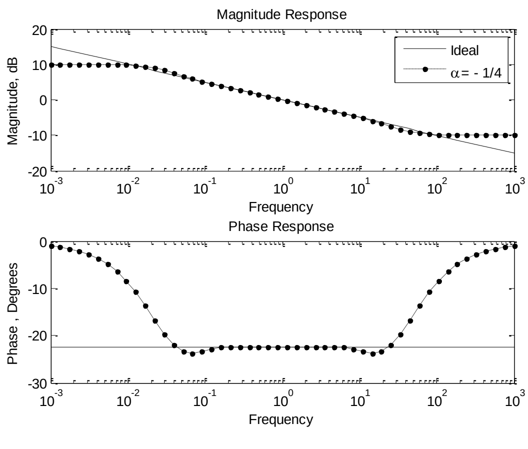

Fig.3.Magnitude and Phase responses for α=-1/4

In this paper, the rational approximations obtained for α=-1/3,-1/4 has been chosen for realization of the Fractance device.

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3. REALIZATION USING PARTIAL FRACTION EXPANSION

The rational approximations obtained in the previous section need to be realized using the basic elements such as resistor and capacitors. In order to do this a MATLAB built in function ***residue***() is used. As per the residue, the transfer function is expanded as,

$$H(s) = R_a + \frac{1}{sC_b + R_b} + \frac{1}{sC_c + R_c} + ---- \qquad (5)$$

The values of R_a, R_b, C_b, ----- will be calculated by writing a MATLAB programme. For fifth order approximation the number of RC sections will be 5. The passive circuit realization for 5^{th} order transfer function is as shown in Fig.4.

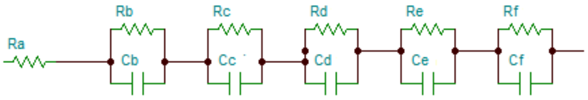

Fig.4.Passive realization of 5^{th} order approximation

The values of the resistances and capacitances for various values of α in steps of 0.1 are tabulated in Table.2.

Table.2.Elements values for various values of fractional order

α	R_a	R_b	C_b	R_c	C_c	R_d	C_d	R_e	C_e	R_f	C_f
0.1	0.6329	0.1849	0.3024	0.1163	2.7561	0.1126	9.3996	0.1535	23.2565	0.3798	61.4173
0.2	0.3986	0.2569	0.2450	0.1970	1.7371	0.2183	5.1342	0.3432	11.1552	1.0947	24.7963
0.3	0.2485	0.2652	0.2652	0.2448	1.4900	0.3100	3.8278	0.5648	7.2878	2.3913	13.4318
0.4	0.1523	0.2394	0.3262	0.2634	1.4742	0.3809	3.3000	0.8080	5.4904	4.7229	8.2238
0.5	0.0909	0.1975	0.4366	0.2569	1.6076	0.4240	3.1414	1.0536	4.5509	8.9771	5.3886
0.6	0.0521	0.1503	0.6302	0.2296	1.9113	0.4325	3.2646	1.2681	4.0999	17.0544	3.6856
0.7	0.0280	0.1043	0.9941	0.1859	2.5063	0.4000	3.7450	1.3956	4.0548	33.6310	2.5950
0.8	0.0133	0.0629	1.7977	0.1302	3.7972	0.3203	4.9649	1.3438	4.6034	73.2658	1.8647
0.9	0.0047	0.0279	4.4048	0.0668	7.8470	0.1883	8.9763	0.9616	7.0677	210.3463	1.3590

The values of resistances and capacitances for α=-1/3, -1/4 are tabulated in Table3.

Table.3.Element values for fractional orders -1/3, -1/4

parameter	α=-1/3	α=-1/4
R_a	0.2115	0.3152
R_b	0.2593	0.2668
C_b	0.2810	0.2496
R_c	0.2541	0.2248
C_C	1.4662	1.5719
R_d	0.3363	0.2663
C_d	3.5968	4.3300
R_e	0.6442	0.4505
C_e	6.5493	8.8094
R_f	3.0219	1.6486
C_f	11.2930	17.8691

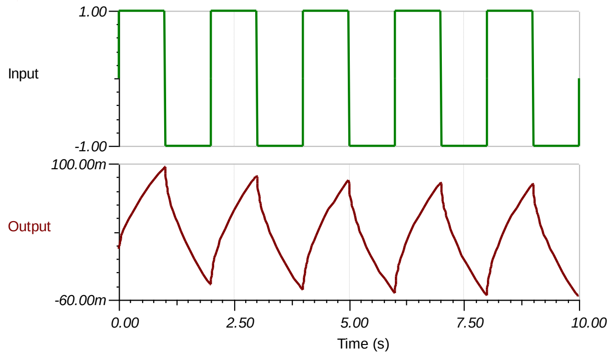

Fig.5.Input and output wave forms of Passive circuit at α=-1/3 for a Square wave at 500mHz

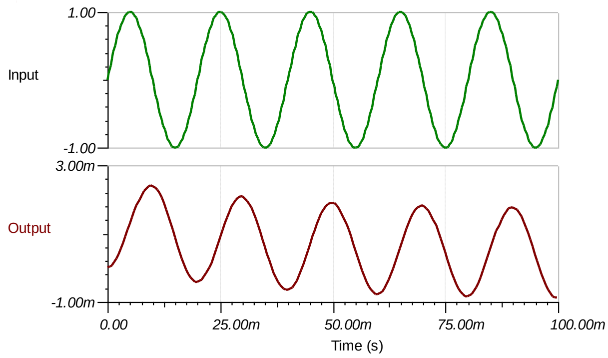

Fig.6.Input and output wave forms of Passive circuit at α=-1/3 for a Sine wave at 50Hz.

4. ACTIVE REALIZATION

The circuit used for the active realization is as shown in Fig.7. Here F stands for Fractance device and two operational amplifiers are used. First operational amplifier provides a phase shift which is nullified to zero when passed through an inverting amplifier connected to it. The value of R_{in} is to be small and R is selected as1KΩ.

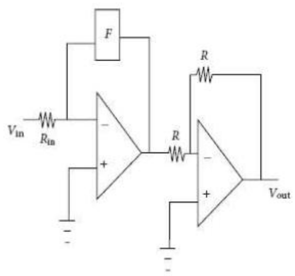

Fig.7.Active realization diagram

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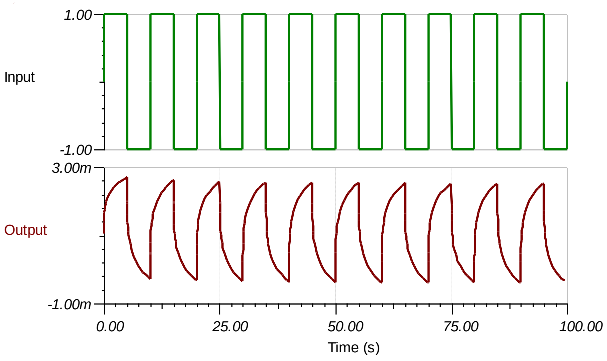

Fig.8.Input and output wave forms of Active circuit at α=-1/4 for a Square wave at 100mHz

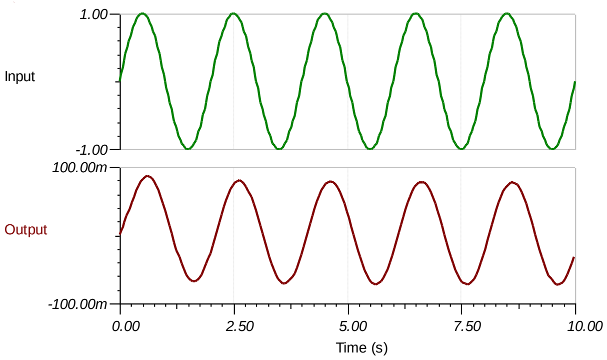

Fig.9.Input and output waveforms of the Active circuit t α=-1/4 for a Sine wave at 500mHz

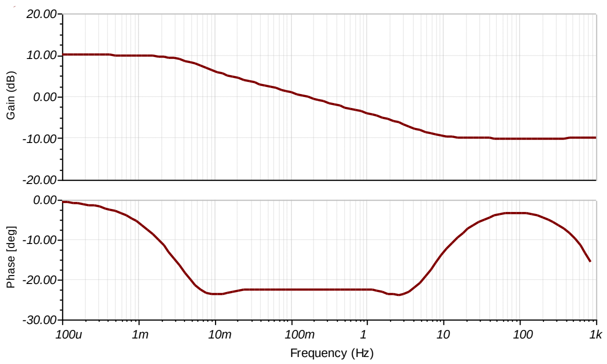

Fig.10.Bode curve of the circuit for α=-1/4

5. CONCLUSIONS

Active and passive realization of the Fractance device for fifth order rational approximation at values of α=-1/3,-1/4 is presented in this paper. Initially by making use of the continued fraction expansion formula the rational approximation for fifth order is calculated. Next, by making use of the MATLAB residue function, the passive circuit is realized. The realized passive circuit is converted into active one by making use of an operational amplifier.

Figs.1-3 present the magnitude , phase responses for the values of fractional order -1/3, -1/4. It can be observed that the Fractance device works good at low frequencies. The outputs of the passive realized circuits is shown in Figs.5 and 6. From the graphs it is evident that a square waveform is converted into a ramp signal, there is a phase shift for the sinusoidal input. Passive Circuit is converted into active one by making use of an operational amplifier. Figs.8,9 and 10 represents the output wave forms for different excitations and bode plot. Fig.10 is similar to the magnitude and phase response obtained theoretically in Fig.3.It can be observed that the experimental and theoretical results match each other.

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